Experiments with inhomogeneous cross sections

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1 Inhomogeneous cross sections

The material properties of the two isotropic materials are given in Table 1. The orientation of the coordinate system is presented in Figure 1. The

Table 1: Material properties for the two isotropic materials.

Material	Isotropic #1	Isotropic #2
E_{zz}	100	$100/\alpha$
$E_{xx} = E_{yy}$	100	$100/\alpha$
G_{yz}	41.667	$41.667/\alpha$
$G_{xz} = G_{xy}$	41.667	$41.667/\alpha$
ν_{yz}	0.2	0.2
$\nu_{xz} = \nu_{xy}$	0.2	0.2

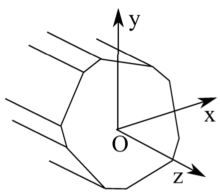

Figure 1: Cross section coordinate system.

relations between strains and curvatures and, forces and moments is given as

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \\ \psi_6 \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \\ M_x \\ M_y \\ M_z \end{bmatrix}$$

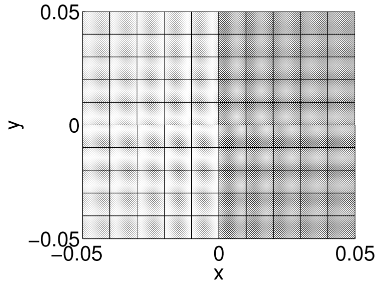

Figure 2: Geometry and finite element mesh of square cross section with two materials S2 – isotropic material #1 (light) and isotropic material #2 (dark).

1.1 Square cross section of two isotropic materials - S2

The solid square geometry is divided in two where each half is made of isotropic material #1 and #2, respectively. The section geometry and material distribution are presented in Figure 2. The variation of the value of the non-zero entries of the cross section stiffness matrix with respect to the stiffness ration E_1/E_2 is presented in Table 2. The estimated positions of the shear and elastic centers are presented in Table 3.

Table 2: Non-zero entries of cross section stiffness matrix for square cross section S2 with respect to E_1/E_2 ration.

E_1/E_2	K_{11}	K_{22}	K_{33}	$K_{44} = K_{55}$	K_{66}	K_{26}	K_{35}
1E+00	3.49E-01	3.49E-01	1.00E+00	8.34E-04	5.91E-04	6.96E-19	-4.32E-18
1E+01	1.28E-01	1.92E-01	5.50E-01	4.59E-04	2.77E-04	-3.93E-03	1.13E-02
1E+02	1.38E-01	1.77E-01	5.05E-01	4.21E-04	2.35E-04	-4.33E-03	1.24E-02
1E+03	1.68E-01	1.75E-01	5.01E-01	4.17E-04	2.31E-04	-4.37E-03	1.25E-02
1E+04	1.73E-01	1.75E-01	5.00E-01	4.17E-04	2.30E-04	-4.38E-03	1.25E-02
1E+05	1.73E-01	1.75E-01	5.00E-01	4.17E-04	2.30E-04	-4.38E-03	1.25E-02

As the stiffness of the weak material vanishes, the stiffness values and positions of shear and elastic centers converge to those which would be obtained if only half the cross section was considered. Note however the variation of the entry K_{11} with respect to the ration E_1/E_2 plotted in Figure 3. The shear stiffness as estimated by VABS is lower than that of half the section and has a minimum at $E_1/E_2 = 10$. The reason for such behaviour is unknown.

1.2 Half-cylinder cross section of one isotropic material – C2

The geometry and finite element mesh are presented in Figure 4. The value of the non-zero entries in the cross section stiffness matrix are presented

Table 3: Shear and elastic center positions ((x_s, y_s) and (x_t, y_t), respectively) for square cross section S2 with respect to the E_1/E_2 ration.

E_1/E_2	x_s	y_s
1E+00	0.000E+00	0.000E+00
1E+01	2.045E-02	0.000E+00
1E+02	2.450E-02	0.000E+00
1E+03	2.495E-02	0.000E+00
1E+04	2.500E-02	0.000E+00
1E+05	2.500E-02	0.000E+00
E_1/E_2	x_t	y_t
1E+00	0.000E+00	0.000E+00
1E+01	2.045E-02	0.000E+00
1E+02	2.450E-02	0.000E+00
1E+03	2.495E-02	0.000E+00
1E+04	2.500E-02	0.000E+00
1E+05	2.500E-02	0.000E+00

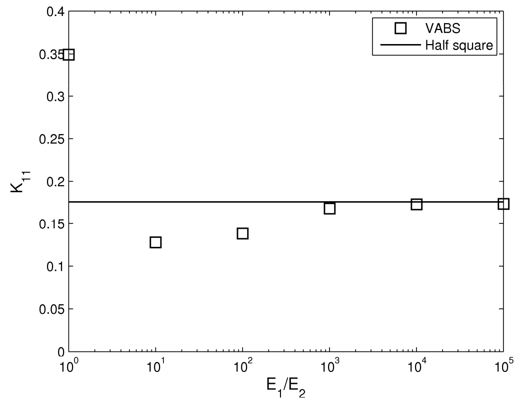

Figure 3: Variation of K_{11} entry of the cross section stiffness matrix with respect to the ration E_1/E_2. Results compared with the solution for half the section.

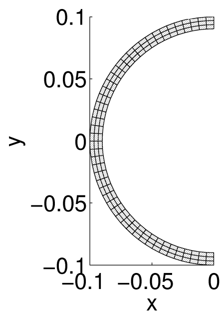

Figure 4: Geometry and finite element mesh of half-cylinder cross section C3.

in Table 4. The estimated positions of the elastic and shear center are presented in Table 5.

Table 4: Non-zero entries of cross section stiffness matrix for half-cylinder cross section C3.

K_{11}	4.964E-02
K_{22}	6.244E-02
K_{33}	2.982E-01
K_{44}	1.349E-03
K_{55}	1.349E-03
K_{66}	9.120E-04
K_{35}	1.805E-02
K_{26}	-7.529E-03

Table 5: Shear and elastic center positions ((x_s, y_s) and (x_t, y_t), respectively) for half-cylinder cross section C4.

x_s	-1.206E-01
y_s	0.
x_t	-6.051E-02
y_t	0.

1.3 Cylinder cross section of two isotropic material (half) – C3

The effect of extreme material inhomogeneity is investigate in the same manner as it was done using the solid square cross section. The cylindrical cross section is divided in two. One half is made of isotropic material #1 while the other half of isotropic material #2 with the properties given in Table 1. The distribution of the material is detailed in Figure 5. The

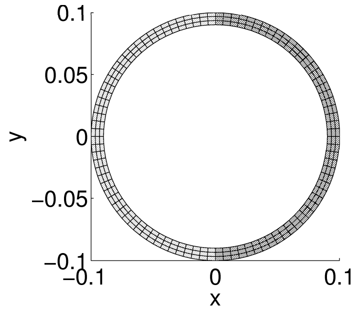

Figure 5: Geometry and finite element mesh of cylinder cross section with two materials – isotropic #1 (light) and isotropic #2 (dark) material.

resulting non-zero entries of the cross section stiffness matrix are given in Table 6 with respect to the ration E_1/E_2. In general, as the stiffness of the isotropic material #2 vanishes the cross section stiffness values converge to those which were obtained when only half the cross section was modelled. Note however the behaviour of the K_{11} entry with respect to the ration E_1/E_2 plotted in Figure 6. Just like in the case of the solid square cross section, the shear stiffness seems to decrease past the half-cylinder values having a minima at $E_1/E_2 = 10$. Once again the reason for this behavior cannot be explained. Finally the variation of the positions of the shear and elastic centers with respect to the ration E_1/E_2 are presented in Table 7. Also, note that the positions of the shear and elastic center do not explain the behavior of the K_{11} entry.

Table 6: Non-zero entries of cross section stiffness matrix for cylinder cross section C2 with respect to E_1/E_2.

E_1/E_2	K_{11}	K_{22}	K_{33}	K_{44}	$K_{55} = K_{55}$	K_{34}	K_{16}
1E+00	1.25E-01	1.25E-01	5.96E-01	2.70E-03	2.25E-03	-1.94E-18	1.76E-17
1E+01	3.99E-02	6.87E-02	3.28E-01	1.48E-03	1.08E-03	6.78E-03	1.62E-02
1E+02	3.75E-02	6.31E-02	3.01E-01	1.36E-03	9.29E-04	7.45E-03	1.79E-02
1E+03	4.74E-02	6.25E-02	2.99E-01	1.35E-03	9.14E-04	7.52E-03	1.80E-02
1E+04	4.94E-02	6.24E-02	2.98E-01	1.35E-03	9.12E-04	7.53E-03	1.80E-02
1E+05	4.96E-02	6.24E-02	2.98E-01	1.35E-03	9.12E-04	7.53E-03	1.80E-02

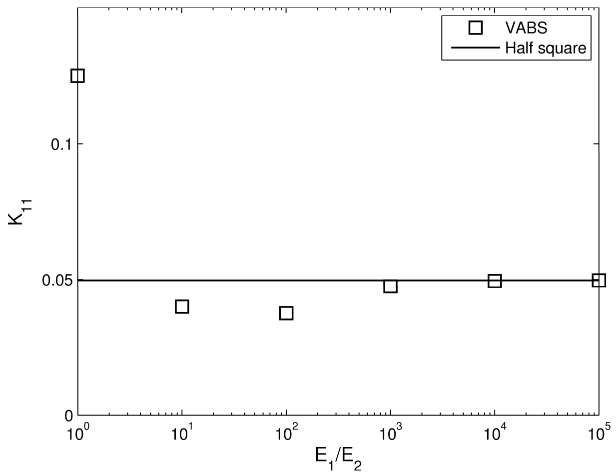

Figure 6: Variation of K_{22} entry of the cross section stiffness matrix with respect to the ration E_1/E_2. Results compared with the solution for half cylinder.

Table 7: Shear and elastic center positions ((x_s, y_s) and (x_t, y_t), respectively) for cylinder cross section C2 with respect to the ration E_1/E_2.

E_1/E_2	x_s	y_s
1E+00	0.000E+00	0.000E+00
1E+01	-9.866E-02	-1.069E-15
1E+02	-1.182E-01	-1.415E-15
1E+03	-1.203E-01	-1.292E-15
1E+04	-1.206E-01	3.740E-16
1E+05	-1.206E-01	1.308E-15
E_1/E_2	x_t	y_t
1E+00	0.000E+00	0.000E+00
1E+01	-4.951E-02	3.279E-17
1E+02	-5.931E-02	1.920E-17
1E+03	-6.039E-02	3.411E-18
1E+04	-6.050E-02	4.960E-18
1E+05	-6.051E-02	-1.255E-18